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Control of the vorticity mode in the linearized Euler equations for hybrid aeroacoustic prediction

C. Prax [a,*], F. Golanski [b], L. Nadal [a]

[a] *Laboratoire d'Etudes Aérodynamiques, Université de Poitiers, 40, avenue du Recteur Pineau, 86022 Poitiers, France*
[b] *Department of Mechanical and Materials Engineering, McLaughlin Hall, Queen's University, Kingston, Ontario, Canada K7L 3N6*

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Abstract

The issue of vorticity mode perturbation in Linearized Euler Equations (LEE) is addressed in this paper. We chose to tackle this question by the point of view of source term formulation. It is numerically shown that the use of a rotational free acoustic source term significantly reduces the development of the hydrodynamic mode. In accordance with the theory, the proposed source term lead to a quasi total absence of vorticity mode in a spatially uniform mean flow, and a strong reduction in a sheared mean flow.

Keywords: Computational aeroacoustics; Linearized Euler equations; Vorticity mode

1. Introduction

The question of noise generated by turbulent flows is a major issue in many industrial applications. In the transport industry, the reduction of noise and enhancement of sound comfort constitutes a commercial and economic stake of foreground; one can evoke the civil aircrafts design which have to reduce the noise impact on the population living by the airports; there is also several military situations where the control of noise is crucial for safety or tactical reasons.

The mechanisms of noise generation are about known but they involve the fine scales of turbulence. The engineering challenge in this domain is the question of predictability of the sound emitted by a flow in a given situation. An important research activity has emerged in the development of efficient numerical tools for the simulation of aeroacoustics. The prediction of what are the modifications of the radiated noise induced by slight changes in the flow parameters or in the device design is still an important challenge.

The Navier–Stokes equations describe the behavior of a compressible fluid in a complete and exact way and constitute for that the system to be solved to obtain simultaneously the dynamic and the acoustic solutions of a

* Corresponding author.
E-mail address: christian.prax@lea.univ-poitiers.fr (C. Prax).

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problem defined by boundary and initial conditions. The huge power of modern computers makes possible to compute a DNS (for direct numerical simulation). One can then solve the whole range of scales involved in the flow. This leads to "exact solutions" that can be considered as reference solutions. However this very powerful technique is almost exclusively practicable within a research context because of its very high cost. The idea of separately solving the aerodynamic (or hydrodynamic) problem and the acoustic propagation has been introduced by Lighthill [1] in a famous article that marked the beginning of the modern aeroacoustic era. It has since inspired other hybrid approaches and splitting techniques where the acoustic solution is given by a forced wave operator, the forcing being modeled from the solution of the dynamic problem. For a detailed review of the issues of computational aeroacoustics we recommend the recently published review article by Wang et al. [2].

Low Mach number flows constitute a real challenge for DNS. Indeed, for these flows, the characteristic scales related to the flow dynamic and those related to the generated sound are very different. Then, computing both of them in the same simulation (which is done in DNS) inevitably lead to the following issue: an error that would be of acceptable magnitude in the flow study can be of the same order as the sound generated by this flow. In such a situation, an hybrid approach is particularly suited [2]. For these flows, a one-way coupling of the aerodynamic and acoustic phenomena can be undertaken by the use of incompressible assumption. The large disparity in the scales is no longer problematic, and efficient numerical tools can be used at each step.

Up to now, the Lighthill analogy has been extensively used. Indeed, its formulation into a simple wave equation and its resolution by the Green's function makes it very appealing as well in the experimental investigations as in the numerical developments. Nevertheless, the Lighthill equation cannot clearly distinguish the source generation phenomena from the so called propagation effects. This issue is theoretically not the main one in the low Mach number flow framework when the base flow for the source computation is a compressible simulation. However, in the case where the first step is an incompressible simulation, the Lighthill analogy cannot account for the propagation effects in the source terms [3,4]. In this case, only an explicit formulation of these terms can describe it. This is what was sought for in the analogies formulated by [5,6]. Whereas the Phillips equation does not take all the propagation effects, the Lilley equation can account for the refraction and convection effects in a non-uniform medium. The main difficulty posed by the Lilley equation is its difficult numerical resolution. More precisely, the solution to the Lilley equation contains homogeneous spatially growing instability waves that become unbounded in transversally sheared mean flow.

More recently, the use of the linearized Euler's equations (also called LEE in the remaining) has become very popular: They can account for the propagation effects while allowing to distinguish the source generation and the propagation parts. Moreover, their linear features make them attractive in a theoretical point of view as well as for their numerical resolution.

Numerically, there are still some problems posed by the LEE. The first issue is linked to the fact that the LEE can not only deal with acoustic waves in a non-uniform medium but also with the so called entropy and vorticity modes. These last two fluctuating modes are convected by the mean flow. In a real flow, these fluctuations are normally limited by the non-linearities of the motion equation and by viscosity. Some difficulties arise in the LEE since their natural damping by non-linearities is no longer present in the solved equations. As a consequence they can grow exponentially and perturb the acoustic solution. This problem corresponds to the one encountered in the Lilley equation (it has been shown by [7] that in the transversally sheared mean flow, the LEE and the Lilley equations are equivalent). They can either disturb the solution or make the simulation unstable.

Several techniques have been employed by previous authors to deal with this issue. The first attempts was to add some non-linear terms in the LEE to avoid the exponential growth [8]. This works fine but reduces the efficiency of the global method: the use of non-linear terms increases the computational cost (by increasing the spatial and temporal resolution requirement) and complexity of the resolution [9]. Another solution proposed by [7,10] was to use a reduced operator. Indeed, it was shown that the growth of the instability can be associated with the interaction between acoustic waves and gradients of the mean flow. The reduced operator is the LEE operator without these mean flow gradients. This leads to a reduction of the amplifications but is still too weak in some strong cases to be able to perform a correct noise prediction. More recently, there was some attempts to reformulate completely the wave operator such that it completely removes the vorticity mode from the final solution. The acoustic perturbation equations of [11] is one, but its implementation reaches a high level of complexity in the case of incompressible flows. In the same spirit [12] have proposed the LPCE (for linearized perturbed compressible equations). Their purpose is the same as the APE, say they define a new wave operator which is completely vorticity mode free. Other authors, [13], proposed an analytical method to suppress the instability waves. Their approach completely removes these waves, but they require to solve the problem directly in the frequency domain. Then their solution cannot be applied to time-domain resolutions.

In the present paper, we propose to make benefit of the natural formulation of the acoustic sources to reduce considerably the impact of the vorticity mode. By "natural formulation" we mean the source formulation arising by the formal coupling between the Low Mach Number Approximation (called LMNA in the following) and the LEE. In the next section, we will recall the different motion equations used in this study (LMNA and LEE) and quickly explain how they lead to a source term formulation. We also explain why they allow to reduce considerably the growing of the vorticity mode. In the third section, we apply these source terms and compare them to the classical quadrupole expression, in the academic test case of two co-rotating vortices.

2. Model equations

Following the numerical approach adopted for the simulation of non-isothermal flows by [14] for subsonic regime, the low Mach number approximation was used to establish the set of equations for the dynamic solution. This solution is free of the compressibility constraint and is valid for the simulation of inhomogeneous flows where the local density depends only on temperature or inhomogeneities effects. For isothermal flows, this set of equations is equivalent to the classical set of incompressible Navier–Stokes equations. In the precedent studies, the aeroacoustic development strategy showed that the linearized Euler equations can be derived with their own sources formulation. In this study we focus on the effect of this source term definition on the acoustic and vorticity response of linearized Euler equations.

In this section we develop the reasoning that lead from the dynamic (LMNA) equation to the propagation step (LEE) revealing new formulations for acoustic sources.

Since the readers interested in the topic of this paper might not be familiar with the LMNA approach, a detailed presentation of this model is provided in the following paragraph. We intend here to facilitate the understanding of the whole physical significance of the source formulation that we discuss here.

2.1. The low Mach number approximation

The low Mach number approximation we are using was initially developed for reacting flows by [15,16]. Being free of acoustic waves, this approximation exhibits the same advantages for hybrid method as an incompressible assumption: the computational requirements associated with multi-scale features are absent for this model.

To obtain the mathematical formulation of the low Mach number approximation, we start with the fully compressible Navier–Stokes equations written for a perfect gas flow, without heat sources. These equations, written for non-dimensional quantities take the form:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_j}{\partial x_j} = 0 \tag{1}$$

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re}\frac{\partial \tau_{ij}}{\partial x_j} \tag{2}$$

$$\frac{\partial E}{\partial t} + \frac{\partial (p+E)u_j}{\partial x_j} = \frac{1}{Re}\frac{\partial u_i \tau_{ij}}{\partial x_j} + \frac{1}{M^2 Re Pr(\gamma-1)}\frac{\partial^2 T}{\partial x_j^2} \tag{3}$$

$$p = \frac{\rho T}{\gamma M^2} \tag{4}$$

where $(x_1, x_2, x_3) = (x, y, z)$ are the cartesian coordinates and $(u_1, u_2, u_3) = (u, v, w)$ are the velocity components. p, ρ, T are, respectively, the pressure, density and temperature. The total energy per unit volume E and the viscous stress tensor τ are, respectively, written

$$E = \frac{p}{\gamma - 1} + \frac{1}{2}\rho u_i u_i \tag{5}$$

where the conventional summation for repeated index is used and

$$\tau_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij} \tag{6}$$

The dimensional references are U_{ref}, T_{ref}, L_{ref}, ρ_{ref}, v_{ref}. In accordance, the time reference is $L_{\text{ref}}/U_{\text{ref}}$. The pressure reference is $\rho_{\text{ref}}U_{\text{ref}}^2$. Re and Pr are respectively the Reynolds and Prandtl numbers based on the corresponding reference quantities. The Mach number is defined as $M^2 = U_{\text{ref}}^2/(\gamma r T_{\text{ref}})$, where γ is the ratio of specific heat at constant pressure and volume, and r is the universal gas constant.

The LMNA is obtained by an asymptotic development of the compressible Navier–Stokes equations for the vanishing parameter $\varepsilon = \gamma M^2$. The development in ε is defined by the following relations

$$\rho = \rho^{(0)} + \varepsilon\rho^{(1)} + \cdots \tag{7}$$

$$u_i = u_i^{(0)} + \varepsilon u_i^{(1)} + \cdots \tag{8}$$

$$T = T^{(0)} + \varepsilon T^{(1)} + \cdots \tag{9}$$

To maintain the coherence of the whole model, the pressure is developed from the perfect gas law (4) in which we use the Eqs. (7) and (9):

$$p = \frac{\rho^{(0)}T^{(0)}}{\varepsilon} + \rho^{(0)}T^{(1)} + \rho^{(1)}T^{(0)} + \cdots$$

which is used in the following as

$$p = \frac{p^{(0)}}{\varepsilon} + p^{(1)} + \cdots \tag{10}$$

In the following, development is stopped at this stage for p. This slightly different expansion allows us to keep a physical interpretation of the resulting equations coherent with the reference quantities that allow this expansion. The dimensional references for $p^{(0)}$ and $p^{(1)}$ are, respectively, $\rho_{\text{ref}}T_{\text{ref}}$ and $\rho_{\text{ref}}U_{\text{ref}}^2$. Following the terminology of [16], this leads to a physical interpretation of $p^{(0)}$ as a thermodynamic pressure, whereas $p^{(1)}$ will be referred to as a dynamic pressure.

When substituting the flow variables by their expansion expressions (7)–(10) in the compressible Navier–Stokes equations (1)–(4), we obtain independent systems of equations corresponding to their order in ε. They have to be verified independently at each order, and its only when comparing the result to an actual flow at a given Mach number that a finite value of ε provides their relative importance.

The lowest order for these equations is ε^{-1} for the momentum equation and the energy equation. For this order the momentum equation is reduced to

$$\frac{\partial p^{(0)}}{\partial x_i} = 0 \quad \forall i = 1,2,3 \tag{11}$$

This equation shows that $p^{(0)}$, which we recall is to be interpreted as a thermodynamical pressure will be uniform in space. This term can be time dependant if we consider a closed domain [17] or stationary if we consider an open domain, as it will always be the case in the present study.

The equations governing the flow dynamic are obtained at the order ε^0 and ε^{-1}:

$$\frac{\partial\rho^{(0)}}{\partial t} + \frac{\partial\rho^{(0)}u_i^{(0)}}{\partial x_i} = 0 \tag{12}$$

$$\frac{\partial\rho^{(0)}u_i^{(0)}}{\partial t} + \frac{\partial\rho^{(0)}u_i^{(0)}u_j^{(0)}}{\partial x_j} = -\frac{\partial p^{(1)}}{\partial x_i} + \frac{1}{Re}\frac{\partial\tau_{ij}^{(0)}}{\partial x_j} \tag{13}$$

$$\rho^{(0)}\frac{\partial u_i^{(0)}}{\partial x_i} = \frac{1}{RePrT^{(0)}}\frac{\partial^2 T^{(0)}}{\partial x_j^2} \tag{14}$$

$$p^{(0)} = \rho^{(0)}T^{(0)} \tag{15}$$

This system is close to a classical incompressible assumption. It can be viewed as an incompressible assumption where the density variations related to non-isothermal effects (described by the perfect gas law (15) and the energy conservation equation (14)) are allowed to act on momentum and mass conservation equations (13) and (12). When the initial flow field is isothermal, this system is exactly equivalent to the classical divergence-free incompressible assumption.

Numerically, this system behaves like a classical incompressible assumption in the sense that, compared to the compressible system, the equation for the conservation of energy is no longer an equation of evolution. Then, $p^{(1)}$, as well as an incompressible pressure, acts as a Lagrange multiplier, and has to be obtained via the resolution of a Poisson equation. The details of the numerical procedure for the resolution of this system are beyond the scope of the present article, and would not give any insight in the present analysis. Readers interested by this last point should direct to [14] where the exact numerical implementation is detailed, and to [16,17] for a general appreciation of the numerical specificities of this approach.

To name the basics of the resolution, spatial derivatives are estimated by sixth-order compact finite-difference schemes of [18] and the time integration is performed by an explicit fourth-order Runge–Kutta scheme. Both of them are frequently employed in Computational AeroAcoustics (CAA) and have proved their adequacy with the requirements of aeroacoustics [2].

The next paragraph shows that the linearized Euler equations can be obtained from the development at the order ε^1, using classical assumptions.

2.2. Path from the LMNA to the linearized Euler's equations

The development of equations at the upper order ε^1 are

$$\frac{\partial\rho^{(1)}}{\partial t}+\frac{\partial}{\partial x_j}(\rho^{(0)}u_j^{(1)}+\rho^{(1)}u_j^{(0)})=0 \tag{16}$$

$$\frac{\partial\rho^{(0)}u_i^{(1)}}{\partial t}+\rho^{(1)}\frac{\partial u_i^{(0)}}{\partial t}+(\rho^{(0)}u_j^{(1)}+\rho^{(1)}u_j^{(0)})\frac{\partial u_i^{(0)}}{\partial x_j}+\frac{\partial\rho^{(0)}u_j^{(0)}u_i^{(1)}}{\partial x_j}=\frac{1}{Re}\frac{\partial\tau_{ij}^{(1)}}{\partial x_j} \tag{17}$$

$$\frac{\partial p^{(1)}}{\partial t}+\frac{\partial}{\partial x_j}(p^{(1)}u_j^{(0)}+\gamma p^{(0)}u_j^{(1)})+(\gamma-1)p^{(1)}\frac{\partial u_j^{(0)}}{\partial x_j}=\frac{\gamma-1}{Re}\tau_{kj}^{(0)}\frac{\partial u_k^{(0)}}{\partial x_j}+\frac{\gamma}{RePr}\frac{\partial^2T^{(1)}}{\partial x_j^2} \tag{18}$$

Up to now, no other assumption than the low Mach number assumption has been made. The model systems, correspond simply to different order of asymptotic development. In the following, we will introduce the classical assumptions that are usually made in CAA when using the linearized Euler's equations.

In the set of equations (16)–(18) $(\rho^{(0)}, u_i^{(0)}, p^{(0)})$ are instantaneous flow fields given by the resolution of the CFD equations (12)–(15). In order to have a good estimation of the propagation effect (in the sense of the terminology of [2]) of a non-uniform flow, it is generally sufficient to consider a steady mean flow associated with the actual flow. By considering the propagation over a steady mean flow, this set of equations can be modified by replacing instantaneous flow fields by their time average quantities (ρ_0, u_{0i}, p_0). In this case, the presence of the term $\rho^{(1)}\frac{\partial u_i^{(0)}}{\partial t}$ is no longer justified in (17). Acoustic waves traveling along a short distance being hardly affected by viscous effects, it is also justified to drop the viscous terms $\frac{\gamma-1}{Re}\tau_{kj}^{(0)}\frac{\partial u_k^{(0)}}{\partial x_j}+\frac{\gamma}{RePr}\frac{\partial^2T^{(1)}}{\partial x_j^2}$ in (18), and $\frac{1}{Re}\frac{\partial\tau_{ij}^{(1)}}{\partial x_j}$ in (17). At this stage, two differences remain with the LEE system [19]. First the term $-(\gamma-1)u_j^{(1)}\frac{\partial p^{(0)}}{\partial x_j}$, present in the LEE, is absent in Eq. (18). However, it was shown that $p^{(0)}$ is uniform. Secondly, the gradient of perturbed pressure does not appear in (17) while it is present in the corresponding equation in the LEE system. Then, modifying (17) while adding on both sides $\frac{\partial p^{(1)}}{\partial x_i}$ allows us to recover the LEE

$$\frac{\partial\rho_0u_i^{(1)}}{\partial t}+(\rho_0u_j^{(1)}+\rho^{(1)}u_{0j})\frac{\partial u_{0i}}{\partial x_j}+\frac{\partial\rho_0u_{0j}u_i^{(1)}}{\partial x_j}+\frac{\partial p^{(1)}}{\partial x_i}=\frac{\partial p^{(1)}}{\partial x_i} \tag{19}$$

Now, the classical LEE for a small perturbation (ρ', u_i', p') over a steady mean flow (ρ_0, u_{0i}, p_0), can be identified by replacing in the left hand side of equations (16, 19 and 18) $(\rho^{(1)}, u_i^{(1)}, p^{(1)})$ with the perturbations (ρ', u_i', p'). The change in notations we are doing in this part between $(\rho^{(0)}, u_i^{(0)}, p^{(0)})and(\rho_0, u_{0i}, p_0)$ on one hand, and $(\rho^{(1)}, u_i^{(1)}, p^{(1)})and(\rho', u_i', p')$ on the other hand is of paramount importance. Indeed, all the superscript $\cdot^{(0)}$and $\cdot^{(1)}$ refer to the asymptotic development of the LMNA. They are free of physical assumption, and in the theoretical limit where the development is continued to an infinite order, we could get from these values the real compressible flow. On the other hand, the subscript $\cdot_0$ and prime $\cdot'$ refer to the conventional superposition of a mean flow and perturbed quantities.

This distinction being kept in mind, $p^{(1)}$ in the left hand side of (19) is to be associated with the perturbed quantity p', whereas in the right hand side, it is to be associated with the solicitation, coming from the actual instationnarity of the CFD solution, generating noise and vorticity. In other words, in the right hand side, $p^{(1)}$ has to keep its full asymptotic development meaning.

Finally, the acoustic production and propagation can be obtained in the present context by solving the LEE:

$$\frac{\partial \rho'}{\partial t} + \frac{\partial}{\partial x_j}(\rho_0 u_j' + \rho' u_{0j}) = 0 \tag{20}$$

$$\frac{\partial \rho_0 u_i'}{\partial t} + (\rho_0 u_j' + \rho' u_{0j})\frac{\partial u_{0i}}{\partial x_j} + \frac{\partial \rho_0 u_{0j} u_i'}{\partial x_j} + \frac{\partial p'}{\partial x_i} = S_i \tag{21}$$

$$\frac{\partial p'}{\partial t} + \frac{\partial}{\partial x_j}(p' u_{0j} + \gamma p_0 u_j') + (\gamma - 1)\left[p'\frac{\partial u_{0j}}{\partial x_j} - u_j'\frac{\partial p_0}{\partial x_j}\right] = 0 \tag{22}$$

It is well established now that the linearized Euler equations can represent accurately the acoustic propagation in a non-uniform medium by taking into account the refraction and convection effects. Following the terminology of [2] these effects will now be called propagation effects.

The expression of S_i is one of the keys of the acoustic analogy. The present development gives the following forcing term definition to be considered as the acoustic sources

$$S_i = \frac{\partial p^{(1)}}{\partial x_i} \tag{23}$$

This result have to be linked to the Ribner theory of dilatation [20] who proposes the incompressible pressure of turbulent flows to predict noise. He shows that the effective production of sound is the fact of the unsteady dilatation of fluid elements, driven by inertial effects. The far acoustic field generated by a quasi incompressible flow is obtained by considering an equivalent medium at rest containing a spatial distribution of sound sources. The pressure perturbation in the flow region is split into ‘‘pseudosound’’ and ‘‘acoustic’’’ pressure. The pseudosound pressure is associated to inertial forces in the flow and satisfy the incompressible Poisson equation for the pressure. The dilatation equation is obtained by subtracting the Poisson equation to the Lighthill equation written in terms of total pressure perturbation. Ribner virtual source terms is obtained by virtue of isentropic consideration, and would be expressed from the incompressible pressure as $-\frac{1}{c^2}\frac{\partial^2 p^{(1)}}{\partial t^2}$ in our case. However, in the form of a single wave equation, the corresponding source term of (23) is $-\nabla^2 p^{(1)}$. This term is similar to the one that derives from the coupling between the LMNA and the Lighthill analogy in [14]. In Eq. (21), p' is not split and is the total pressure perturbation, i.e. pseudosound and sound in the flow source region. Moreover, in the context of a LMNA simulation of the dynamic, the definition (23) is available to predict sound both in isothermal and non-isothermal situations.

By considering the momentum equation of the LMNA system (13), we can write

$$S_i = -\frac{\partial \rho^{(0)} u_i^{(0)}}{\partial t} - \frac{\partial \rho^{(0)} u_i^{(0)} u_j^{(0)}}{\partial x_j} + \frac{1}{Re}\frac{\partial \tau_{ij}^{(0)}}{\partial x_j} \tag{24}$$

If we neglect the viscous source term $\frac{1}{Re}\frac{\partial \tau_{ij}^{(0)}}{\partial x_j}$ – as usually assumed in the case of flow evolving far from solid boundaries [1] – S_i is finally expressed as

$$S_i \simeq \underbrace{-\frac{\partial \rho^{(0)} u_i^{(0)}}{\partial t}}_{S_i^t} \underbrace{-\frac{\partial \rho^{(0)} u_i^{(0)} u_j^{(0)}}{\partial x_j}}_{S_i^q} \tag{25}$$

This source term is consistent with the one defined in [21,22]. Indeed, the divergence of S_i^t is zero in the isothermal situations and does not affect the acoustic mode. In isothermal case, expression (25) thus reduces to S_i^q, the quadrupole source term originally derived by Lighthill. Also defined in [22,10], it contains the shear noise and the self noise and is sufficient for isothermal flows. On the acoustic radiation point of view, anyone of the source (23) or S_i^q can serve as the vehicle for predicting noise without changing the quadrupolar nature of source. Conversely, in non-isothermal flows, S_i^t is not divergence-free, so it contributes to noise radiation. The entire form (25) of S_i is then required.

While limiting ourselves to experimentation in isothermal situations, we are interested in the following sections to the behavior of the LEE solutions when employing forcing terms S_i (23) estimated directly from the incompressible pressure data and S_i^q (25).

3. Vorticity mode and source term analysis

The general solution of the LEE contains three different modes of perturbation. This decomposition can be related to the theory of the modes of perturbation introduced by [23] in the general case of viscous flows.

The behavior of each of these modes in the general solution depends on the nature of the mean flow, and on the nature of the source terms. In the general situation of a non-uniform mean flow, a lot of cases were reported where the vorticity mode (sometimes called hydrodynamic mode, or instability wave) leads to a non-physical solution, or to an unstable simulation. Indeed, these three modes are really present in the flow. Then each of them has to be well represented in a DNS. The vorticity mode is a convective mode which grows downstream of the region where it is created. In a real flow, its growth is limited by non-linear interactions and viscous effects. With an hybrid method, the interactions between these different modes and the evolution of each of them taken separately can be slightly different than in a physical flow such as modeled by a DNS. The vorticity mode is very well represented by the first step of the hybrid method (here the LMNA simulation) but the non-linear terms and the viscous effects are absent in the second step. It explains why the growth of this term in space and time is physical in a real flow but unphysical in the propagation step where in extreme cases, it can make the simulation unstable.

Several attempts have been made by previous authors to circumvent this issue. In the past, authors have dealt with this issue by modifying the propagation operators in such a way to control or prohibit the development of the vorticity mode [8,7,10–12].

All these works consider that the issue results from the fact that the solution of the LEE contains the vorticity fluctuations. This is true, but we rather think that the difficulties arise when the vorticity fluctuations are over-excited by improper source terms. By improper source terms we mean source terms that are properly designed to excite the acoustic mode but excessively excite the vorticity mode.

Here we are considering the use of S_i defined in (23) to fix this problem. We will show that by using S_i, which is an irrotational field, the vorticity mode observed in the numerical resolutions is dramatically reduced. The use of the complete LEE is possible with appearance of contained and limited vorticity modes in numerical solutions.

The presence of the vorticity mode in the LEE solution can be allotted to two different processes. The vorticity can be either created by the source term itself or can result from interactions between any other mode (acoustic, entropic) and a non-uniform mean flow. Subsequently, if the mean flow is uniform, the vorticity results only from the source term itself; If the source term has an irrotational formulation, it cannot excite the vorticity mode and there will be no vorticity created in the flow. However, in the general case of a non-uniform mean flow, the vorticity mode can result both from an excitation coming from the formulation of the source terms which is not irrotational and from interactions between the mean flow and other modes of fluctuation. Thus, in the situation where the rotational of the source term is controlled to zero, the only vorticity generation in the solution of the LEE will result from modes interactions in non-uniform mean flow.

The source term S_i^q (25), deriving from the velocity components computed from an incompressible DNS, does not have reasons to be rotational free, while S_i, by definition (23), is rotational free. As a consequence, driving the LEE with S_i instead of S_i^q should limit the presence of vorticity mode. Given that S_i could be evaluated by (24), or by (25) when neglecting viscous terms, and given that the divergence of S_i^t is null (if the density is constant), the divergences of $S_i$andS_i^q are precisely the same. According to what was written in this section, it is clear that driving the LEE with S_i instead of S_i^q can reduce one of the major difficulty inherent to the LEE linked to the presence of the vorticity mode. Besides, the acoustic solution should not be altered in any way since the divergence of the pressure gradient is exactly the same as the divergence of the source term previously used by other authors. Rather than to calculate S_i using velocity components (25), it is of course more effective to compute straight from the hydrodynamic pressure as it was done in the following numerical tests.

4. Numerical results

As mentioned above, the LMNA equations are equivalent to the incompressible equations when the density is supposed constant. This allows to use S_i as well as S_i^q to define acoustic source terms on the solution obtained with an incompressible Navier–Stokes solver. We recall that S_i^q solutions have been validated with direct acoustic computation in situations where vorticity generation was not critical [24]. Some numerical simulations have been carried out to illustrate the response of LEE in density and vorticity. The sound field and the vorticity field resulting from LEE solution are then compared.

4.1. Sound radiated by the pairing of two co-rotating vortices

An incompressible direct numerical simulation is achieved to simulate the flow consisting in the motion of co-rotating vortices. The configuration of the flow shown in Fig. 1 was presented by [25]. We use the model of Scully vortex, where the distance between the two vortices at the beginning is $2r_0 = 20$ and the radius $r_c = 2$ in this all part, L_{ref} is chosen so that $r_c = 2L_{\text{ref}}$ and $U_{\text{ref}} = c_0$ (where c_0 is the speed of sound). The Mach number M of the flow based on the maximum tangential velocity of the vortex is $M = 0.5$.

The dimension of the numerical box is $Lx \times Ly$ with a uniform grid consisting in $nx \times ny$ nodes. Sixth-order compact scheme was used and third order Runge–Kutta scheme with time increments of $\mathrm{d}t = 0.6$. Two sizes of numerical domain were tested: 140×140 with 320×321 nodes and 240×240 with 600×601 nodes.

Fig. 2 shows four instantaneous fields of vorticity obtained with the incompressible simulation, from the beginning of the simulation to an instant when the vortices are merged.

The results of the flow simulation, i.e., the instationary velocity components, are used to determine the forcing terms S_i^q while S_i is given by the pressure gradient. One on four time steps and one on two nodes in both directions are retained in the LEE simulation. A larger numerical domain is considered to get the acoustic fluctuations away from the source region which is centered in this domain. In all the cases presented here, the acoustic region extent is $Lx' \times Ly' = 1000 \times 1000$. We now consider several situations with no mean flow, a uniform flow, and a sheared mean flow and discuss the behavior of the solution for each solicitation.

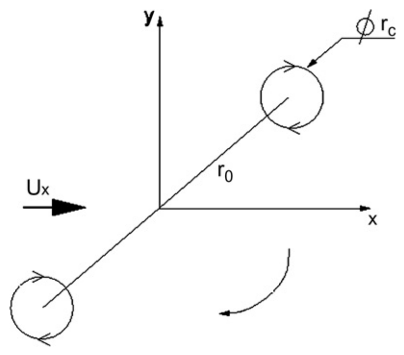


Fig. 1. Flow configuration for the pairing of two co-rotating vortices.

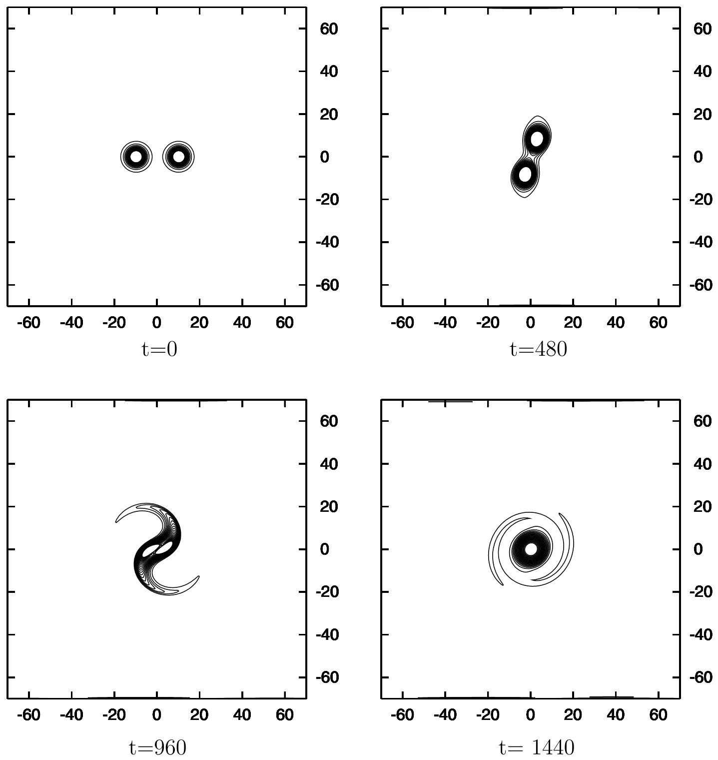


Fig. 2. Vorticity field obtained with the incompressible simulation at four different instants on the smaller domain (140 × 140). Levels are from −0.12 to −0.005 by step 0.005.

4.1.1. *Case 1: no mean flow*

With this case, we first analyze the dependency on the size of the source domain and the truncation error for both source terms S_i and S_i^q. Indeed, S_i is known to decrease rather slowly compared to S_i^q [26]. Thus, a higher sensitivity to the size of the source domain is expected for S_i. Fig. 3 shows the density field at the same instant with S_i and S_i^q.

The source domain size is exactly the size of the DNS domain. The small domain show relatively different patterns with S_i and S_i^q. A close look at the figure corresponding to S_i (top right of Fig. 3) shows pronounced lobes that suggest a truncation of the source term in a region where it is not completely vanished. This would confirm the well known argument against the use of pressure as an acoustic source term for incompressible flows. The extended domain presents the closer pattern with S_i^q.

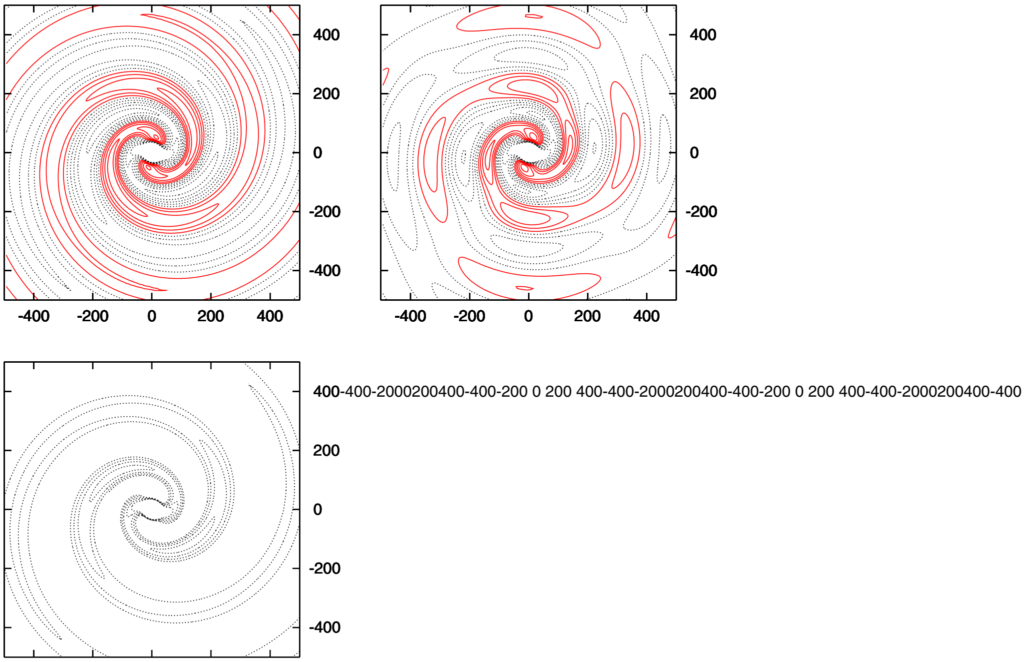
This is confirmed by instantaneous profiles of the density along a line at $y = 200$ presented in Fig. 4. On the large domain, S_i and S_i^q provided very similar profiles in shape (Fig. 3) and amplitude (Fig. 4). This difference in amplitude can be considered small enough for the purpose of the present investigation in which we focus on the control of the vorticity mode.

Provided that the source domain is large enough, it is then possible to use the incompressible pressure gradient as forcing term to lead to a correct acoustic solution.

4.1.2. *Case 2: uniform mean flow $u_x = 0.5$*

The computed sources are now used as forcing terms for LEE with mean flow condition set to a constant value $u_x = 0.5$. Fig. 5 represents for each source term the density in the acoustic region and the vorticity in the source region. According to part 3 the created vorticity is vastly reduced (even not visible at this scale) when source term S_i is used.

The acoustic fields are very similar. We can also notice that in the case where vorticity is created at the vortices location, this vorticity does not increase while convected downstream. This is consistent with the theory



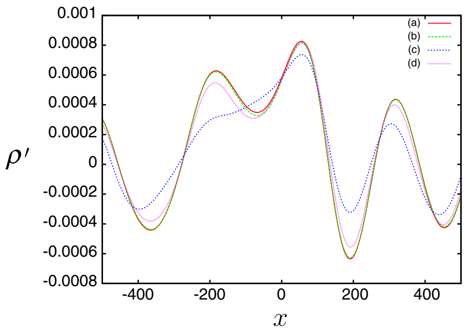
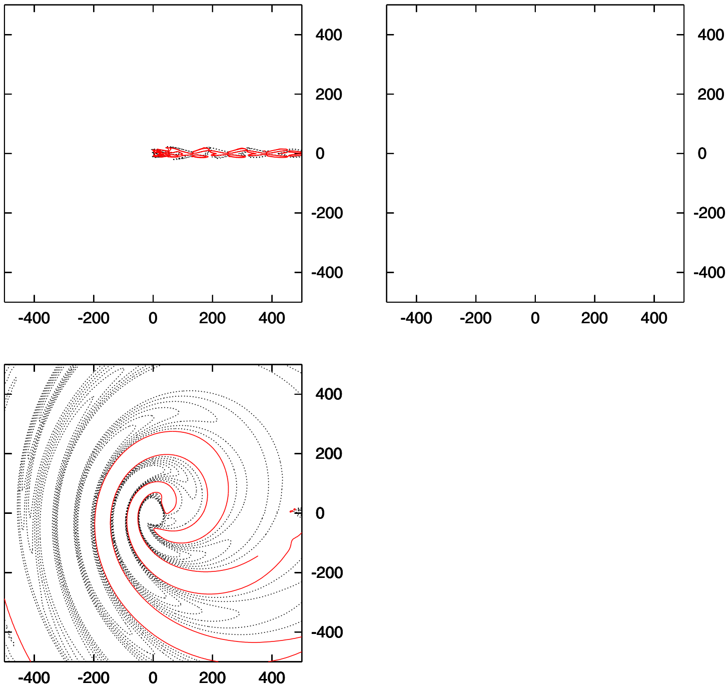


Fig. 4. Density ρ' along a line $y = 200$ at $t = 1152$ obtained with S_i^q and S_i calculated on two sizes of source domain. S_i^q: (a) 140 × 140; (b) 240 × 240. S_i: (c) 140 × 140; (d) 240 × 240.

of which elements have been exposed above. This case is not considered as very constraining for the simulation robustness, however, the small amplitude vorticity generated by S_i^q is large enough to put the outflow boundary condition of [27] into troubles (region close to $x = 500$, $y = 0$ of the bottom left graph of Fig. 5). The next case is designed to observe what happen in sheared flow cases, which are known to lead to the exponential growth of the vorticity mode.

4.1.3. Case 3: sheared mean flow

Since the flow was either null or uniform in the two precedent cases, there was no creation of vorticity from the interaction between the acoustic field and the mean flow. To see what happen in this situation, the sources terms are now used as forcing terms in solving LEE in a non-uniform mean flow

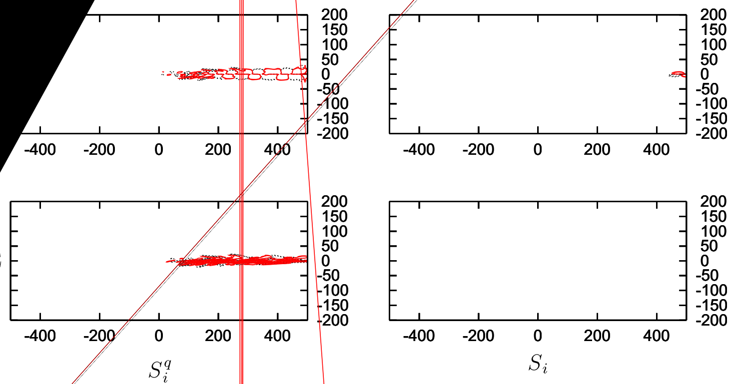


$$u_x(y) = \frac{U_1 + U_2}{2} + \frac{U_1 - U_2}{2} \tanh\left(\frac{2y}{\delta_0}\right)$$

U_1 and U_2 are, respectively, the velocity in the upper and lower region and δ_0 stands for the vorticity thickness. In the present test, they are chosen to be $U_1 = 0.5$ and $U_2 = 0.25$.

Fig. 6 shows a snapshot of the density in presence of this sheared mean flow. This figure shows the results provided by the complete set of LEE with S_i and S_i^q. It also shows the results provided by the reduced LEE [7] with the different source formulations. On the density field, large fluctuations appears downstream of the actual acoustic source materialized by the vortex pairing. We can identify this region as a non-physical region where the created vorticity interacts with the mean flow to create spurious density fluctuations and acoustic sources. They are present in both cases, but the use of S_i strongly attenuate these fluctuations level. When using the reduced operator, for each source formulation, the density produced by interactions with the vortical mode is reduced compared to the full LEE. While with S_i^q, neither the full LEE nor the reduced LEE allow to evacuate the vortical mode at the outlet, the combination of S_i and the reduced operator lead to spectacular attenuation. There is no visible sound generation downstream of the pairing and consequently no trouble with the outlet: there is no vortical mode to evacuate.

The vorticity fields obtained in these four cases (Fig. 7) confirm that S_i significantly reduces the vorticity amplitude. The two cases computed with the source term S_i^q (top and bottom left in Fig. 7) present a high concentration of vorticity emitted at the center of the domain and convected downstream. The full LEE computed with the term S_i (top right in Fig. 7) presents a smaller concentration of vorticity, which moreover seems to be produced only further downstream. The reduced LEE with term S_i, presents no visible vorticity mode at all. To reveal more precisely the vorticity content of the solutions obtained with S_i, Fig. 8 shows the vorticity isocontours at one tenth of the levels presented in the previous figure. Vorticity is created at the very center of the vortex roll up and convected downstream when full LEE are used, and at the levels presented here, no more visible vorticity is convected downstream of the source centre when the reduced LEE are associated with



n flow: vorticity field obtained with the two sources formulations and for the full or reduced LEE operator. Values are with a step of 0,005. Straight lines: $\omega_z' > 0$; dashed line $\omega_z' < 0$.

S_i. To summarize, the intensity of the non-physical perturbation on the density field presented in re very well correlated with the intensity of the vorticity generated in the different cases in Figs. 7 The drastic improvement from the full LEE associated with the source term S_i^q to the reduced LEE iated with S_i appears directly related to the better control of the vorticity mode.

hese three different cases show that significant improvements can be obtained without modifying the LEE perator but only the source term definition. In the case of the sheared mean flow it is required to use the educed operator, but again, the full source formulation is the only one that can ensure the reliability of the solution when the perturbations are convected through the outlet.

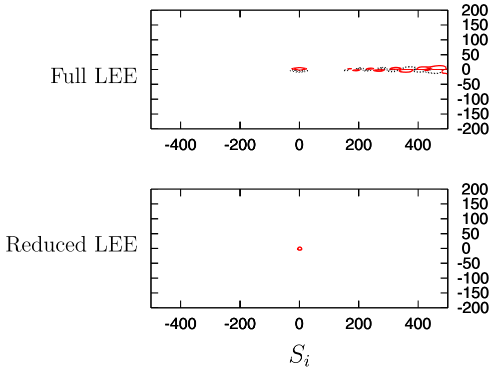


Fig. 8. Sheared mean flow: vorticity field obtained with S_i for the full or reduced LEE operator. Values are one tenth of Fig. 7 from -0.001 to 0.001 with a step of 0.0005. Straight lines: $\omega_z' > 0$; dashed line $\omega_z' < 0$.

5. Conclusion

In this exploring study, numerical simulations show that it should be possible to use the gradient of incompressible pressure as acoustic source term in the context of solving linearized Euler equations. This choice can significantly reduce the problems caused by the generation and amplification of vorticity mode in sheared flows. This source term is rotational free. An academic test case has been designed to prove the efficiency of this approach. Drawback of non-compactness of dynamic pressure fluctuations induced by more compact acoustic stress field led us to oversize the source domain and to envisage more elaborated source truncation strategies not developed in this paper.

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